

Solutions to Mathematics Workbook-1 | Quadratic Equations & Inequalities [2022]

JEE Main (Archive)	Daily Tutorial Sheet 2
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11.(A) $x + y = -\frac{y}{2x}$

$$\therefore -\frac{y}{2x} + \frac{x}{y} = \frac{1}{2} \Rightarrow a - \frac{1}{2a} = \frac{1}{2} \Rightarrow 2a^2 - 1 = 1a$$

$$\Rightarrow 2a^2 - a - 1 = 0 \Rightarrow 2a^2 - 2a + a - 1 = 0$$

$$\Rightarrow (a-1)(2a+1) = 0$$

$$a = 1, -\frac{1}{2}$$

$$a = \frac{x}{y} > 0 \text{ as } x < 0, y < 0$$

$$\therefore a = 1 \Rightarrow x = y$$

12.(B) Since, $a, b, c > 0$ and $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$

Case-I: $D \geq 0$ i.e., Roots are real

$$\frac{C}{a} > 0 \Rightarrow \text{Roots have same sign}$$

$$-\frac{b}{a} < 0 \Rightarrow \text{Roots are negative.}$$

Case-II: $D < 0$. Let the 2 roots be $\alpha \pm i\beta$

$$-\frac{b}{a} = 2\alpha \Rightarrow \alpha < 0$$

$\Rightarrow$ Both roots have negative real part.

13. Given, $2^{|y|} - |2^{y-1} - 1| = 2^{y-1} + 1$

Case I When $y \in (-\infty, 0]$ $\therefore 2^{-y} + (2^{y-1} - 1) = 2^{y-1} + 1$

$$\Rightarrow 2^{-y} = 2 \Rightarrow y = -1 \in (-\infty, 0] \quad \dots (i)$$

Case II When $y \in (0, 1]$

$$\therefore 2^y + (2^{y-1} - 1) = 2^{y-1} + 1 \Rightarrow 2^y = 2$$

$$\Rightarrow y = 1 \in (0, 1] \quad \dots (ii)$$

Case III When $y \in (1, \infty)$

$$\therefore 2^y - 2^{y-1} + 1 = 2^{y-1} + 1 \Rightarrow 2^y - 2 \cdot 2^{y-1} = 0 \Rightarrow 2^y - 2^y = 0 \text{ true for all } y > 1 \quad \dots (iii)$$

From Eqs. (i), (ii) and (iii), we get

$$y \in \{-1\} \cup [1, \infty).$$

Alternate Solution:

Case-I: $y > 1 \Rightarrow 2y - 2^{y-1} + 1 = 2^{y-1} + 1$

$$\Rightarrow 2^y = 2^y. \text{ Always true.}$$

$$\text{Case-II: } y \leq 1 \Rightarrow 2^{|y|} + 2^{y-1} - 1 = 2^{y-1} + 1$$

$$\Rightarrow 2^{|y|} = 2 \Rightarrow |y| = 1 \Rightarrow y = \pm 1$$

$$14. \text{ Given, } \frac{2x}{2x^2 + 5x + 2} > \frac{1}{x+1} \Rightarrow \frac{2x}{(2x+1)(x+2)} - \frac{1}{(x+1)} > 0 \Rightarrow \frac{2x(x+1) - (2x+1)(x+2)}{(2x+1)(x+2)(x+1)} > 0$$

$$\Rightarrow \frac{-(3x+2)}{(2x+1)(x+1)(x+2)} > 0; \text{ using number line rule}$$

$$\begin{array}{ccccccc} & - & & + & & - & & + & & - \\ & | & & | & & | & & | & & | \\ -2 & & -1 & & -\frac{2}{3} & & -\frac{1}{2} & & \end{array}$$

$$\therefore x \in (-2, -1) \cup \left(-\frac{2}{3}, -\frac{1}{2}\right)$$

$$15. \text{ Given, } (5+2\sqrt{6})^{x^2-3} + (5-2\sqrt{6})^{x^2-3} = 10 \dots (i)$$

$$\text{Put } y = (5+2\sqrt{6})^{x^2-3} \Rightarrow (5-2\sqrt{6})^{x^2-3} = \frac{1}{y}$$

$$\text{From Eq. (i), } y + \frac{1}{y} = 10 \Rightarrow y^2 - 10y + 1 = 0 \Rightarrow y = 5 \pm 2\sqrt{6}$$

$$\Rightarrow (5+2\sqrt{6})^{x^2-3} = 5+2\sqrt{6} \text{ or } (5+2\sqrt{6})^{x^2-3} = 5-2\sqrt{6}$$

$$\Rightarrow x^2 - 3 = 1 \text{ or } x^2 - 3 = -1 \Rightarrow x = \pm 2 \text{ or } x = \pm\sqrt{2} \Rightarrow x = \pm 2, \pm\sqrt{2}$$

$$16. \text{ The given equation can be rewritten as } \frac{2}{\log_a x} + \frac{1}{\log_a ax} + \frac{3}{\log_a a^2 x} = 0 \quad [\because b = a^2 x, \text{ given}]$$

$$\Rightarrow \frac{2}{\log_a x} + \frac{1}{1 + \log_a x} + \frac{3}{2 + \log_a x} = 0 \Rightarrow \frac{2}{t} + \frac{1}{1+t} + \frac{3}{2+t} = 0, \text{ where } t = \log_a x$$

$$\Rightarrow 2(1+t)(2+t) + 3t(1+t) + t(2+t) = 0 \Rightarrow 6t^2 + 11t + 4 = 0$$

$$\Rightarrow (2t+1)(3t+4) = 0 \Rightarrow t = -\frac{1}{2} \text{ or } -\frac{4}{3} \therefore \log_a x = -\frac{1}{2} \text{ or } \log_a x = -\frac{4}{3}$$

$$17.(B) \text{ If } a_1 x^2 + b_1 x + c_1 = 0 \text{ and } a_2 x^2 + b_2 x + c_2 = 0$$

Have a common real root, then

$$\Rightarrow \left. \begin{aligned} (a_1 c_2 - a_2 c_1)^2 &= (b_1 c_2 - b_2 c_1)(a_1 b_2 - a_2 b_1) \quad \therefore \begin{cases} x^2 + bx - 1 = 0 \\ x^2 + x + b = 0 \end{cases} \text{ have a common root.} \end{aligned} \right\}$$

$$\Rightarrow (1+b)^2 = (b^2+1)(1-b) \Rightarrow b^2 + 2b + 1 = b^2 - b^3 + 1 - b$$

$$\Rightarrow b^3 + 3b = 0 \therefore b(b^2 + 3) = 0 \Rightarrow b = 0, \pm\sqrt{3}i$$

$$18.(B) \text{ As we know, } ax^2 + bx + c > 0 \text{ for all } x \in R, \text{ iff } a > 0 \text{ and } D < 0.$$

$$\text{Given equation is } x^2 + 2ax + (10 - 3a) > 0, \forall x \in R$$

$$\text{Now, } D < 0$$

$$\Rightarrow 4a^2 - 4(10 - 3a) < 0 \Rightarrow 4(a^2 + 3a - 10) < 0 \Rightarrow (a+5)(a-2) < 0$$

$$\Rightarrow a \in (-5, 2)$$

19.(A) Let $f(x) = x^2 - 2ax + a^2 + a - 3$

Both roots are less than 3.

$$\Rightarrow \alpha < 3, \beta < 3 \Rightarrow \text{Sum} = \alpha + \beta < 6 \quad \Rightarrow \quad \frac{\alpha + \beta}{2} < 3 \quad \Rightarrow \quad \frac{2a}{2} < 3$$

$$\Rightarrow a < 3 \quad \dots (i)$$

Again, $D = B^2 - 4AC \geq 0$

$$\Rightarrow (-2a)^2 - 4 \cdot 1(a^2 + a - 3) \geq 0$$

$$\Rightarrow 4a^2 - 4a^2 - 4a + 12 \geq 0$$

$$\Rightarrow -4a + 12 \geq 0 \Rightarrow a \leq 3 \quad \dots (ii)$$

Again, $af(3) > 0$

$$\Rightarrow 1[(3)^2 - 2a(3) + a^2 + a - 3] > 0 \Rightarrow 9 - 6a + a^2 + a - 3 > 0 \Rightarrow a^2 - 5a + 6 > 0$$

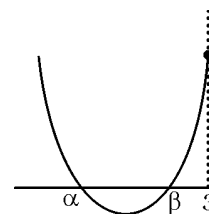
$$\Rightarrow (a - 2)(a - 3) > 0$$

$$\therefore a \in (-\infty, 2) \cup (3, \infty) \quad \dots (iii)$$

From Eqs. (i), (ii) & (iii), we get

$$a < 2$$

Therefore, (a) is the answer.



20. Since, $x^2 - 3x + 2 > 0$ and $x^2 - 2x - 4 \leq 0$

$$\Rightarrow (x - 1)(x - 2) > 0 \text{ and } x^2 - 2x + 1 \leq 5 \Rightarrow (x < 1 \text{ or } x > 2) \text{ and } (1 - \sqrt{5} \leq x \leq 1 + \sqrt{5})$$

$$\therefore x \in [1 - \sqrt{5}, 1) \cup (2, 1 + \sqrt{5}]$$

21. Given, $|x - 2|^2 + |x - 2| - 2 = 0$

Case I When $x \geq 2$

$$\Rightarrow (x - 2)^2 + (x - 2) - 2 = 0 \Rightarrow x^2 + 4 - 4x + x - 2 - 2 = 0 \Rightarrow x^2 - 3x = 0$$

$$\Rightarrow x(x - 3) = 0 \Rightarrow x = 0, 3 \quad [0 \text{ is rejected}]$$

$$\Rightarrow x = 3 \quad \dots (i)$$

Case II When $x < 2$

$$\Rightarrow \{-(x - 2)\}^2 - (x - 2) - 2 = 0 \Rightarrow (x - 2)^2 - x + 2 - 2 = 0 \Rightarrow x^2 + 4 - 4x - x = 0$$

$$\Rightarrow x(x - 4) - 1(x - 4) = 0 \Rightarrow (x - 1)(x - 4) = 0$$

$$\Rightarrow x = 1, 4 \quad [4 \text{ is rejected}]$$

$$\Rightarrow x = 1 \quad \dots (ii)$$

Hence, the sum of the roots is $3 + 1 = 4$.

Alternate Solution

$$\text{Given, } |x - 2|^2 + |x - 2| - 2 = 0 \Rightarrow (|x - 2| + 2)(|x - 2| - 1) = 0$$

$$\therefore |x - 2| = -2, 1 \quad [\text{neglecting } -2]$$

$$\Rightarrow |x - 2| = 1 \Rightarrow x = 3, 1 \Rightarrow \text{Sum of the roots} = 4$$

22.(2) Since, $x^2 - 3kx + 2e^{2\log k} - 1 = 0$ has product of roots 7.

$$\Rightarrow 2e^{2\log k} - 1 = 7 \Rightarrow e^{2\log_e k} = 4$$

$$\Rightarrow k^2 = 4 \Rightarrow k = 2 \quad [\text{neglecting } -2]$$

23. $2 + i\sqrt{3}$ is one of the roots $x^2 + px + q = 0$. Then, other root is $2 - i\sqrt{3}$.

$$\Rightarrow -p = 2 + i\sqrt{3} + 2 - i\sqrt{3} = 4 \text{ and } q = (2 + i\sqrt{3})(2 - i\sqrt{3}) = 7 \Rightarrow (p, q) = (-4, 7)$$